MENIIT
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JEE Advanced : Paper-1 (2016)

IMPORTANT INSTRUCTIONS

A. General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at **9:00 am**, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

B. Optical response sheet

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbonless copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
14. Write your name, roll number and code of the examination center, and sign with pen is the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

C. Darkening the bubbles on the ORS

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ⬭ **COMPLETELY**.
17. The correct way of darkening a bubble is as: ⬬
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble

PART-A : PHYSICS

SECTION 1

- This section contains Five questions.
- Each question has Four options (A), (B), (C) and (D). ONLY one of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases.

1. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B*) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Ans. **[B]**

Sol. $eV_0 = \frac{hc}{\lambda_1} - \varphi_1$

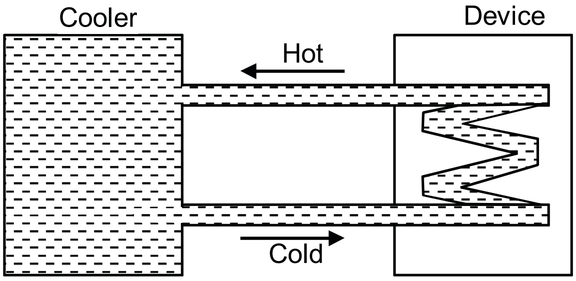
$$V_0 = \frac{hc}{e\lambda_1} - \frac{\varphi_1}{e}$$

$$V_{01} - V_{02} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$h = \frac{e(V_{01} - V_{02})\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

$= 6.4 \times 10^{-34}$ J.s.

2. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B*) 2067 (C) 2533 (D) 3933

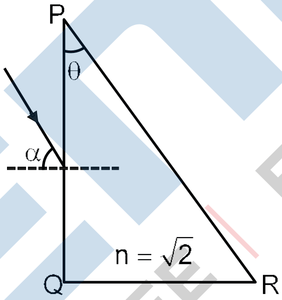
Ans. [B]

Sol. $P = 3000 - \frac{120 \times 4.2 \times 10^3 \times (30-10)}{3 \times 60 \times 60}$

= 3000 – 933

= 2067 watt

3. A parallel beam of light is incident from air at an angle α on the side PQ of a right-angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(A*) 15° (B) 22.5° (C) 30° (D) 45°

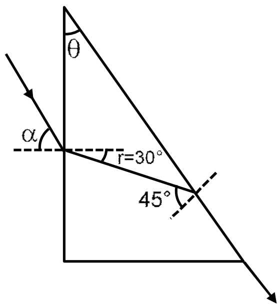
Ans. [A]

Sol. $\sin\theta_c = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$

$1 \sin 45° = \sqrt{2} \sin r$

$\Rightarrow r = 30°$

$\theta = 180° - 45° - 90° - 30° = 15°$



4. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h (< ℓ) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are :

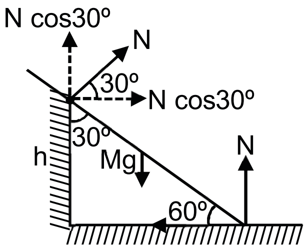
(g = 10 m s^{-2})

(A) $\frac{h}{\ell}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$ (B) $\frac{h}{\ell}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}N$ (C) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}N$ (D*) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$

Ans. **[D]**

Sol. $N\cos 30° = f$

$N + N\sin 30° = Mg$



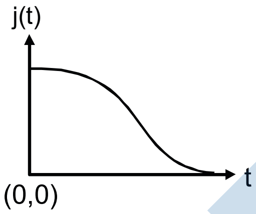
$N = \frac{2mg}{3}$

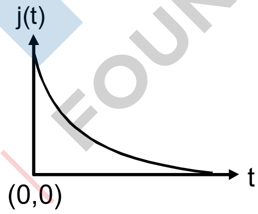
$f = \frac{16\sqrt{3}}{3}N$

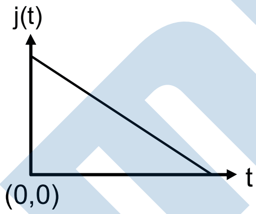
$Mg\frac{\ell}{2}\cos 60° = N\left(\frac{h}{\cos 30°}\right)$

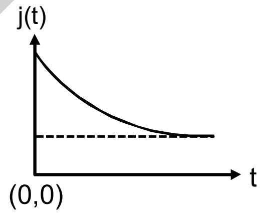
$\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

(A)


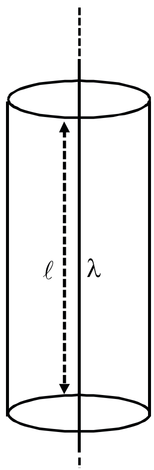
(B*)


(C)


(D)


Ans. **[B]**

Sol. $E = \frac{2k\lambda}{r}$



$j = \sigma E = \sigma\frac{2k\lambda}{r}$

$\frac{i}{2\pi r\ell} = \frac{\sigma 2k}{r}\frac{q}{\ell}$

$i = \frac{\sigma 4\pi q}{4\pi\varepsilon}$

$-\frac{dq}{dt} = \frac{\sigma q}{\varepsilon}$

$$\int_{\lambda\ell}^{q} \frac{-dq}{q} = \int_{0}^{t} \frac{\sigma}{\varepsilon} dt$$

$$-[\ell n\, q - \ell n\, \lambda\ell] = \frac{\sigma t}{\varepsilon}$$

$$\ell n \frac{q}{\lambda l} = \frac{-\sigma t}{\varepsilon}$$

$$q = \lambda\ell\ e^{\frac{-\sigma t}{\varepsilon}}$$

$$i = \frac{-dq}{dt} = \lambda\ell \frac{\sigma}{\varepsilon}\ e^{-\frac{\sigma t}{\varepsilon}}$$

$$j = \frac{\lambda\sigma}{2\pi r\varepsilon}\ e^{-\frac{\sigma t}{\varepsilon}}$$

SECTION 2

- This section contains Eight questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

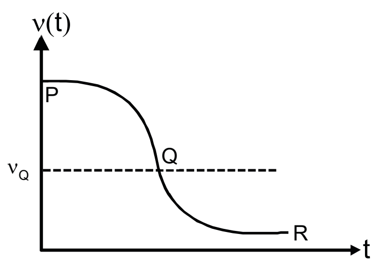
Full Marks	: +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	: +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	: 0 If none of the bubbles is darkened.
Negative Marks	: –2 In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

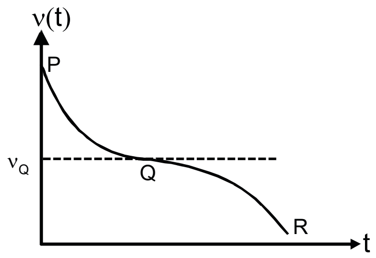
6. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

(A*) The rate of change in beat frequency is maximum when the car passes through Q

(B*) the plot below represents schematically the variation of beat frequency with time



(C*) $\nu_P + \nu_R = 2\nu_Q$



Ans. **[ABC]**

Sol. $f_1 = f_{01}\left(\frac{330 + v\cos\theta}{330}\right)$

$f_2 = f_{02}\left(\frac{330 + v\cos\theta}{330}\right)$

$\Delta f = |f_1 - f_2| = |f_{01} - f_{02}|\left(1 + \frac{v\cos\theta}{330}\right)$

$\Delta f = 3(1 + k\cos\theta)$ $\qquad k = \frac{v}{330}$ constant

7. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C*) The filament emits more light at higher band of frequencies before it breaks up

(D*) The filament consumes less electrical power towards the end of the life of the bulb

Ans. **[CD]**

Sol. For small section of filament

$R = \rho \ell / A$ and A is decreasing due to breakup hence R is increasing with time

For total power consumption

$P = \frac{V^2}{R}$

since V is constant

hence P is inversely proportional to R and due to breakup resistance of different section of the filament increases hence total resistance increases hence total power consumption decreases.

For different section of filament has different resistance due to breakup and $i^2R = eA\sigma T^4$ hence temperature will be different for different section

Just before breakup in the small section of the tungsten filament

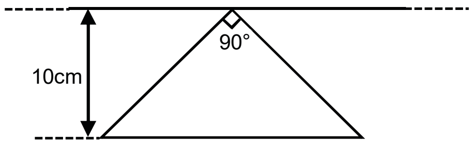
$eA\sigma T^4 = i^2R$

$R \to \infty$

$T \to \infty$

and $\lambda_m T = b$ hence the filament emits more light at higher band of frequencies before it breaks up.

8. A conducting loop in the shape of a right angled isosceles triangle of height 10cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 a s^{-1}. Which of the following statement(s) is(are) true?



(A) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(B) The induced current in the wire is in opposite direction to the current along the hypotenuse

(C*) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(D*) There is a repulsive force between the wire and the loop.

Ans. **[CD]**

Sol. Let current is varying in stright wire and emf is increases in loop (due to reciprocity theorem)

$$d\phi = \int_0^h 2x\,dx \frac{\mu_0 i}{2\pi x}$$

$$\varepsilon = \frac{d\varphi}{dt} = \frac{\mu_0 h}{\pi}\frac{di}{dt} = \frac{\mu_0}{\pi} \times \frac{10}{100} \times 10 = \frac{\mu_0}{\pi}$$

due to rotation there is no change in ϕ

due to lenz law there will be repulsion.

9. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$

where α = 10/3 ms^{-3}, β = 5 ms^{-2} and m = 0.1 kg. At t = 1 s, which of the following statement(s) is (are) true about the particle?

(A*) The velocity $\vec{V}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})ms^{-1}$

(B*) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N m s

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$N

(D*) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}$ N m

Ans. **[ABD]**

Sol. (A) $\vec{r} = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$

$\vec{v} = 3\alpha t^2\hat{i} + 2\beta t\hat{j}$

$v_{t=1s} = 10\hat{i} + 10\hat{j}\, m/s$

(B) $\vec{L} = \vec{r} \times \vec{p}$

$= \frac{-5}{3}\hat{k}\,\text{Nms}$

(D) $\vec{\tau} = \vec{r} \times \vec{F}$

at t = 1s

$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(2\hat{i} + \hat{j}\right) = -\frac{20}{3}\hat{k}\,\text{Nms}$

10. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for ℓ is (are) dimensionally correct?

(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B*) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D*) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

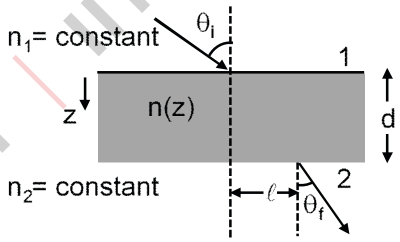
Ans. **[BD]**

Sol. $[\varepsilon] = [M^{-1}L^{-3}T^4A^2]$

$[K_B] = [M^1L^2T^{-2}K^{-1}]$

By dimensional analysis.

11. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($>n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement ℓ.



Which of the following statement(s) is(are) true ?

(A) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

(B*) ℓ is independent of n_2

(C*) $n_1 \sin\theta_i = n_2 \sin\theta_f$

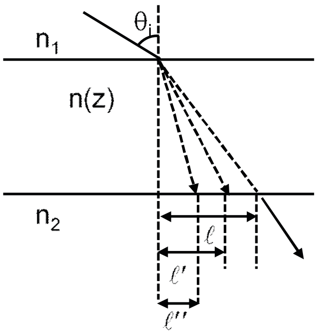
(D*) ℓ is dependent on n(z)

Ans. **[BCD]**

Sol. By snell's law

$\Rightarrow \frac{\sin\theta_i}{\sin\theta_F} = \frac{n_2}{n_1}$

also ⇒ for refraction till the n (z),



⇒ here μ of this medium increases, simultaneosuly, the emergent ray is always depend on the refractive index of medium n (z) (here) also 'ℓ' depends on the refractive index of the medium here n(z) and this ℓ is clearly independent of medium n_2.

12. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true ?

(A*) The refractive index of the lens is 2.5

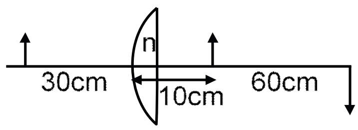
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D*) The focal length of the lens is 20 cm

Ans. **[AD]**

Sol. for mirror (reflection)



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{2}{R}$$

$$\frac{2}{R}=\frac{3-1}{30}=\frac{2}{30}$$

R = 30 cm

For lens (refraction)

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{+60}-\frac{1}{-30}$$

$$=\frac{3}{60}=\frac{1}{20}$$

f = 20

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ (plano convex)}$$

$$\frac{1}{20} = (n-1)\left(\frac{1}{30} - \frac{1}{\infty}\right)$$

$$\frac{1}{20} = \frac{(n-1)}{30}$$

$$n - 1 = \frac{3}{2}$$

$$n = \frac{3}{2} + 1$$

$$= \frac{5}{2} = 2.5$$

13. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is(are) true?

(A*) Relative change in the radii of two consecutive orbitals does not depend on Z.

(B*) Relative change in the radii of two consecutive orbitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D*) Relative change in the angular momenta of two consecutive orbitals varies as 1/n.

Ans. **[ABD]**

Sol. (A) $r = (0.53)\,\dfrac{n^2}{Z}$

$$\ell n\, r = \ell n \frac{(0.53)}{2} + Z\, \ell n\, n$$

(B) $\dfrac{dr}{r} = 0 + Z\,\dfrac{dn}{n}$

(C) $E = 13.6\,\dfrac{z^2}{n^2}$

$$\ell n\, E = \ell n\, 13.6\, Z^2 - 2\, \ell n\, n$$

$$\frac{dE}{E} = \frac{-2dn}{n}$$

(D) $L = \dfrac{nh}{2\pi}$

$$\ell n\, L = \ell n \frac{h}{2\pi} + \ell n\, n$$

$$\frac{dL}{L} = \ell n \frac{h}{2\pi}\frac{dn}{n}$$

SECTION 3

- This section contains five questions.
- The answer to each question is a Single Digit Integer ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

14. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

Ans. **[6]** * Question has dimensional error hc/e should be hc.

Sol. $E = \frac{hc}{\lambda}$

$$E = \frac{1.237\times10^{-6}}{970\times10^{-10}}\,eV\,m = 12.75\,eV$$

+ 12.75 – 13.65 = – 0.85 eV $\Rightarrow$ n = 4

Total energy line = 3 + 2 + 1 = 6

15. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is

Ans. 3

Sol. $V_T = \frac{2gr^2}{9h}(\rho_s - \rho_l)$

$$\frac{V_1}{V_2} = \frac{(8-0.8)}{(8-0.5)}\left(\frac{1}{0.5}\right)^2 \frac{2}{3} = 3$$

16. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

Ans. **[9]**

Sol. $P \propto T^4$

$P = KT^4$

Given $1 = \log_2\left(\frac{K(760)^4}{P_0}\right)$ ... (1)

Let $S = \log_2\left(\frac{K(3040)^4}{P_0}\right)$ ... (2)

$\Rightarrow$ eq. (2) and (1)

$S - 1 = \log_2\left(\frac{3040}{760}\right)^4$

S–1 = 4 × 2 × 1

S = 9

17. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus $\left(^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state? If $^{12}_{5}B$ decays to $^{12}_{6}C^*$, the maximum kinetic energy of the β-particle in units of MeV is (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

Ans. **[8] or [9]**

Sol. $\underset{12.014u}{^{12}_{5}B} \xrightarrow{n \to P} \underset{12u}{^{12}_{6}C} + e^-$

Energy lost = 0.014 × 931.5 = 13.041 MeV

Energy existed = 4.041 MeV

13.041 – 4.041 = 9 MeV

18. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

Ans. **[8]**

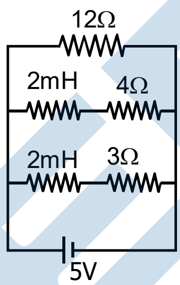
Sol. $$\frac{1}{R_{net}} = \frac{1}{12} + \frac{1}{4} + \frac{1}{3}$$

$$R_{net} = \frac{3}{2}\Omega$$

$$I_{max} = \frac{5 \times 2}{3} = \frac{10}{3}$$

$$I_{min} = \frac{5}{12}$$

$$\frac{I_{max}}{I_{min}} = 8$$



Part B : CHEMISTRY

SECTION 1

19. On complete hydrogenation, natural rubber produces

(A) ethylene-propylene copolymer (B) vulcanised rubber

(C) polypropylene (D) polybutylene

Ans. **[A]**

Sol. Natural rubber is

$-\!\!\left[CH_2 - \underset{\displaystyle CH_3}{\underset{|}{C}} = CH - CH_2\right]_n\!\!-$ on hydrogenation gives ethylene-propylene copolymer

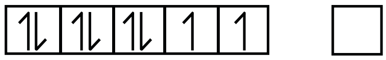
20. Among $[NiCO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **[B]**

Sol. **(i)** $[Ni(CO)_4]$ $sp^3 \rightarrow$ diamagnetic

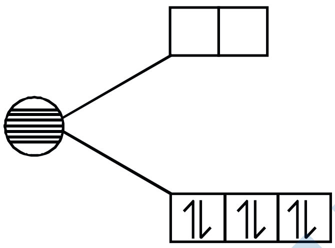
(ii) $[NiCl_4]^{2-}$ $Ni^{2+} \rightarrow (Ar)\ 4s^0 3d^8$



Paramagnetic

(iii) $[Co(NH_3)_4Cl_2]$ Cl

$Co^{+3} \rightarrow 4s^0 3d^6$



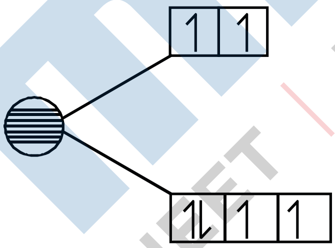
Diamagnetic

(iv) $Na_3[CoF_6]$

↓

WFL

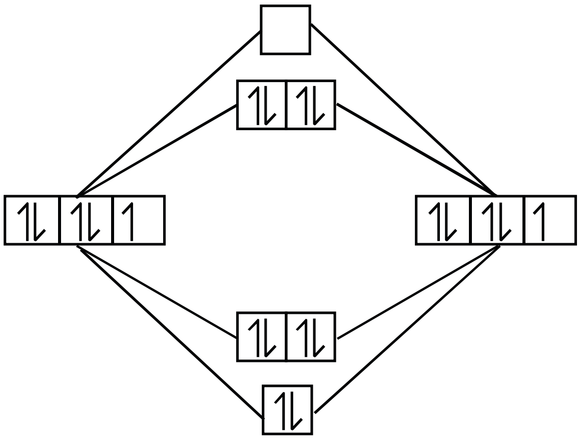
$Co^{+3} \rightarrow 4s^0 3d^6$



Paramagnetic

(v) Na_2O_2

$2Na^{\oplus}$ O_2^{2-}



Diamagnetic

(vi) **$Cs^{+}\ O_2^{-}$**

Paramagnetic

21. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in JK^{-1} is

[1 L atm = 101.3 J]

(A) 5.763 (B) 1.013 (C) –1.013 (D) –5.763

Ans. **[C]**

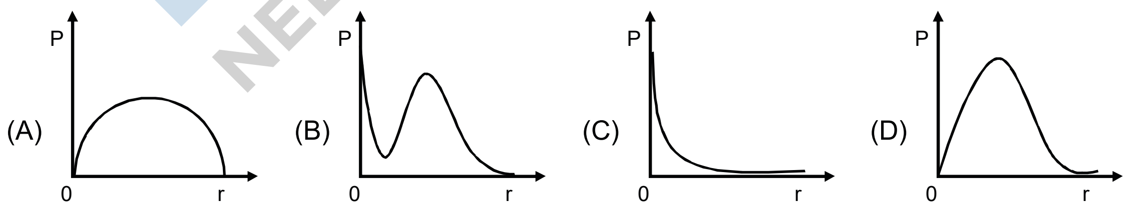
Sol. $\Delta S_{surr} = \frac{-Q_{sys}}{T} = \frac{-(-W_{irr\,isothermal})}{T} = \frac{W_{irr\,isothermal}}{T}$

$W_{irr\ isothermal} = -P_{ext}(V_2 - V_1) = -3(2-1) = -3$ atm lit

$= -3 \times 101.3\ J = -303.9\ J$

$\Delta S_{surr} = \frac{-303.9}{300} = -1.013\ JK^{-1}$

22. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is



Ans. **[D]**

23. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl (B) Ga < Al < In < Tl

(C) Al < In < Ga < Tl (D) Al < Ga < Tl < In

Ans. **[B]**

Sol.

Ga	<	Al	<	In	<	Tl
135		143		167		170 pm

SECTION 2

24. The crystalline form of borax has

(A) tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit

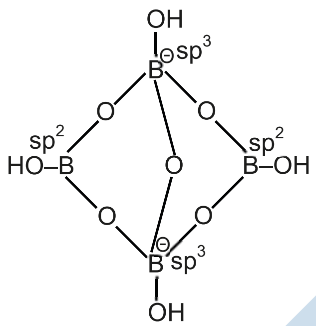
(B) all boron atoms in the same plane

(C) equal number of sp^2 and sp^3 hybridized boron atoms

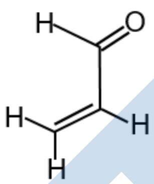
(D) one terminal hydroxide per boron atom.

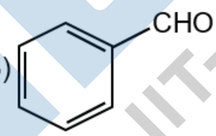
Ans. **[ACD]**

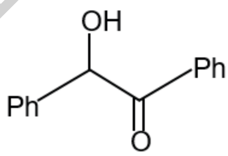
Sol.

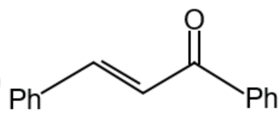


25. Positive Tollen's test is observed for

(A) 

(B) 

(C) 

(D) 

Ans. **[ABC]**

Sol. Tollen's test in given by aldehydes & α-hydroxy ketones.

26. The correct statement(s) about the following reaction sequence is(are)

$$\text{Cumene }(C_9H_{12}) \xrightarrow[\text{(ii) } H_2O^+]{\text{(i) } O_2} P \xrightarrow{CHCl_3/NaOH} Q\text{ (major)} + R\text{ (minor)}$$

$$Q \xrightarrow[PhCH_2Br]{NaOH} S$$

(A) R is steam volatile

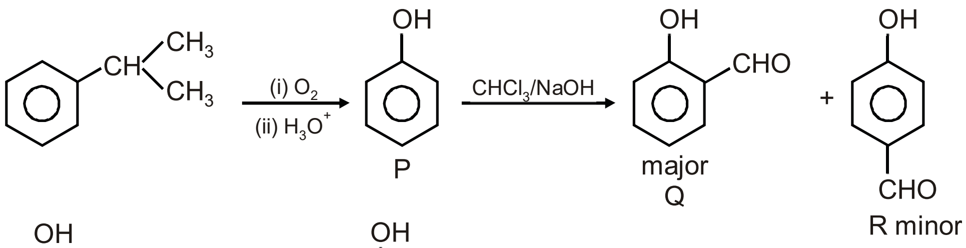
(B) Q gives dark violet coloration with1% aqueous $FeCl_3$ solution

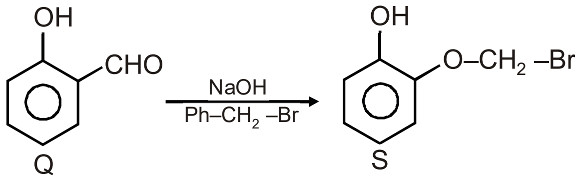
(C) S gives yellow precipitate with 2, 4-dinitrophenylhydrazine

(D) S gives dark violet coloration with 1 % aqueous $FeCl_3$ solution

Ans. **[BC]**

Sol.





S gives yellow precipitate with 2,4–dinitrophyenyl hydrazine.

Q give dark violet coloration with 1% aqueous $FeCl_3$ solution.

27. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is(are)

(A) β^--decay (β emission)

(B) orbital or K-electron capture

(C) neutron emission

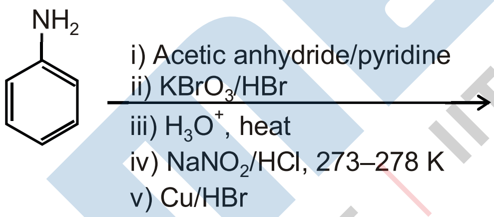
(D) β^+-decay (positron emission)

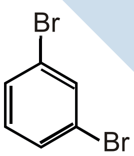
Ans. **[BD]**

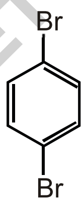
Sol. (B) K-electron capture $\Rightarrow$ p + e^- $\rightarrow$ n

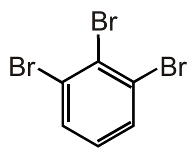
(D) β^+-decay $\Rightarrow$ p $\rightarrow$ n + e^-

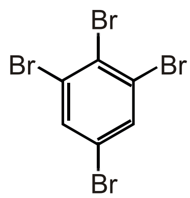
28. The product(s) of the following reaction sequence is(are)



(A) 

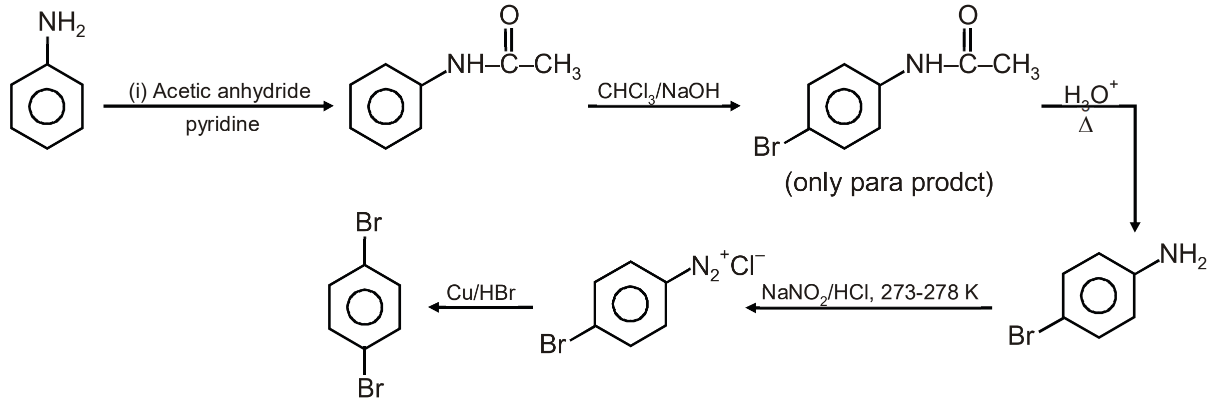
(B) 

(C) 

(D) 

Ans. **[B]**

Sol.



29. According to the Arrhenius equation,

(A) a high activation energy usually implies a fast reaction

(B) rate constant increases with increase in temperature, This is due to a greater number of collisions whose energy exceeds the activation energy.

(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.

(D) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

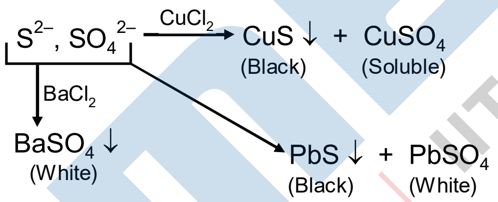
Ans. **[BCD]**

30. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is(are)

(A) $CuCl_2$ (B) $BaCl_2$ (C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

Ans. **[A] or [AC]**

Sol.

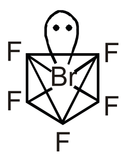
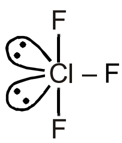
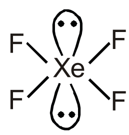
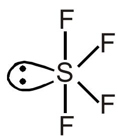


Lead sulphate is soluble in more concentrated solutions of ammonium acetate (6M) or ammonium tartrate (6M) in the presence of ammonia $(NH_4)_2SO_4$

31. The compound(s) with TWO lone pairs of electrons on the central atom is(are)

(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

Ans. **[BC]**

Sol. (A)  (B)  (C)  (D) 

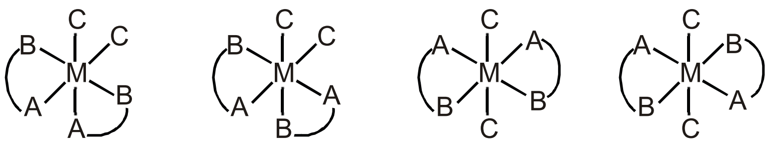
SECTION 3

32. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

Ans. **[5]**

Sol. $[Co(H_2NCH_2CH_2O^-)_2Cl_2]$

$[M(AB)_2C_2]$



33. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is :

Ans. **[9]**

Sol. $x_{solute} = 0.1$ $\quad\quad$ M = m (given)

$x_{solvent} = 0.9$

Assume $n_{solute} = 0.1 \Rightarrow w_{solute} = 0.1 \times M\,Wt_{solute}$

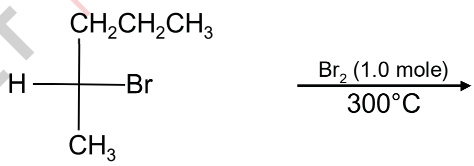
$n_{solvent} = 0.9 \Rightarrow w_{solvent} = 0.9 \times M\,wt._{solvent}$

M = m

$$\frac{0.1\times 1000}{\frac{0.1\times M.Wt._{solute} + 0.9\times Mwt_{solvent}}{2}} = \frac{0.1\times 1000}{0.9\times M.Wt._{solvent}}$$

$$\frac{M.Wt._{solute}}{M.Wt._{solvent}} = 9$$

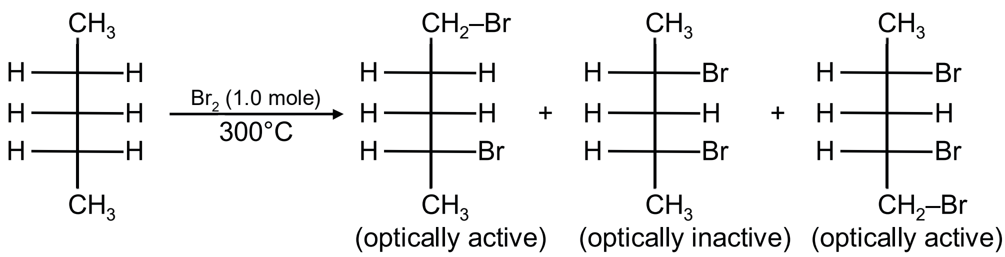
34. In the following monobromination reaction, the number of possible chiral products is

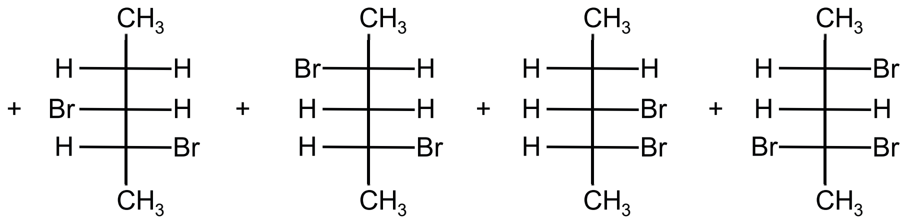


(1.0)
(enantiomerically pure)

Ans. **[5]**

Sol.





(optically active) (optically active) (optically active) (optically inactive)

35. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times, As a result, the diffusion coefficient of this gas increases x times. the value of x is

Ans. **[4]**

Sol. Diffusion coefficient $\alpha\ \lambda \times u_{avg.}$

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2.N^*}$$

$$D.C.\ \alpha\ \frac{u_{avg.}}{\sqrt{2}\pi\sigma^2.N^*}$$

$$N_0 = \frac{P}{KT}$$

$$u_{avg}\ \alpha\ \sqrt{T}$$

$$D.C.\ \alpha\ \frac{T^{3/2}}{P}$$

$$\frac{D.C_2}{D.C_1} = \frac{4\times\sqrt{4}}{2} = \frac{8}{2} = 4$$

36. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is :

Ans. **[6]**

Sol. $MnO_4^- + S_2O_3^{2-} \longrightarrow MnO_2 + SO_4^{2-}$

$n_{factor} = 3$ $\qquad n_{factor} = 4$

Number of equivalent of MnO_4^- = Number of equivalent of SO_4^{2-}

8×3 = mole of $SO_4^{2-} \times 4$

mole of $SO_4^{2-} = 6$

Part C : MATHEMATICS

SECTION 1

37. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

= 10 P (computer turns out to be defective given that it is produced in plant T_2),

where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Ans. [C]

Sol. Let probability of getting defective computer from plant T_1 = x

$\therefore$ probability of getting defective computer from plant $T_2 = \frac{x}{10}$

$\therefore \quad \frac{20}{100} \times x + \frac{80}{100} \times \frac{x}{10} = \frac{7}{100}$

$x = \frac{25}{100} = \frac{1}{4}$

Now, required probability = $\dfrac{\frac{4}{5} \times \frac{39}{40}}{\frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{39}{40}} = \frac{78}{93}$. **Ans.**

38. Let S = $\left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}$ sec x + cosec x + 2 (tan x – cot x) = 0 in the set S is equal to

(A) $\frac{-7\pi}{9}$ (B) $\frac{-2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Ans. **[C]**

Sol. $\sqrt{3}\sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \cos 2x = 0$

$\cos 2x = \cos\left(\frac{\pi}{3} - x\right)$

$2x = 2n\pi \pm \left(\frac{\pi}{3} - x\right)$

for n = 0, $2x = \pm\left(\frac{\pi}{3} - x\right)$ $\Rightarrow$ $x = \frac{\pi}{9}, \frac{-\pi}{3}$

for n = 1, $2x = 2\pi \pm\left(\frac{\pi}{3} - x\right)$

+ve $3x = \frac{7\pi}{3}$ $\Rightarrow$ $x = \frac{7\pi}{9}$ | – ve $2x = 2\pi - \frac{\pi}{3} + x$ $\Rightarrow$ $x = \frac{5\pi}{9}$ (rejected)

for n = – 1, $2x = -2\pi \pm\left(\frac{\pi}{3} - x\right)$

+ve $3x = -2\pi + \frac{\pi}{3} = \frac{-5\pi}{3}$ $\Rightarrow$ $x = \frac{-5\pi}{9}$ | – ve $x = \frac{-7\pi}{3}$ (rejectecd)

$\therefore$ Required sum = $\frac{\pi}{9} - \frac{\pi}{3} - \frac{5\pi}{9} + \frac{7\pi}{9} = 0.$

39. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

Ans. **[A]**

Sol. 4G or 3G + 1B

$\Rightarrow$ $({}^6C_4 + {}^6C_3 \times {}^4C_1) \times {}^4C_1$

$\Rightarrow$ $(15 + 80) \times 4 = 380$. **Ans.**

40. Let $\frac{-\pi}{6} < \theta < \frac{-\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Ans. **[C]**

Sol. $x^2 - 2x\sec\theta + 1 = 0 < \begin{matrix}\alpha_1 \\ \beta_1\end{matrix}$

$x^2 - 2x\sec\theta + \sec^2\theta + 1 - \sec^2\theta = 0$

$(x - \sec\theta)^2 = \tan^2\theta$ $\Rightarrow$ $x - \sec\theta = \pm\tan\theta$ $\Rightarrow$ $x = \sec\theta \pm \tan\theta$

$\because$ $\alpha_1 > \beta_1$

$\therefore$ $\alpha_1 = \sec\theta - \tan\theta,$ $\beta_1 = \sec\theta + \tan\theta,$

Similarly, for the second equation $x^2 + 2x\tan\theta - 1 = 0$

$(x + \tan\theta)^2 = \sec^2\theta$

$x = -\tan\theta \pm \sec\theta$

$\because$ $\alpha_2 > \beta_2$

$\alpha_2 = \sec\theta - \tan\theta, \qquad \beta_2 = -\sec\theta - \tan\theta$

$\therefore \qquad \alpha_1 + \beta_2 = -2\tan\theta.$

41. The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

Ans. **[C]**

Sol. $4\alpha x^2 + \frac{1}{x} \geq 1$

$\alpha \geq \frac{x-1}{4x^3} \quad \forall\, x \in (0, \infty)$

Let $f(x) = \frac{x-1}{4x^3}$

$f'(x) = \frac{1}{4}\left(\frac{x^3\cdot 1-(x-1)\cdot 3x^2}{x^6}\right) = \frac{1}{4}\left(\frac{3-2x}{x^4}\right)$

$\Rightarrow \qquad f(x) \uparrow \text{ in } \left(0, \frac{3}{2}\right) \text{ and } f(x) \downarrow \text{ in } \left(\frac{3}{2}, \infty\right)$

$f(x)\big|_{max.} = f\left(\frac{3}{2}\right) = \frac{1}{27}$

$\therefore$ Least value of α is $\frac{1}{27}$. **Ans.**

SECTION 2

42. Consider a pyramid OPQRS located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

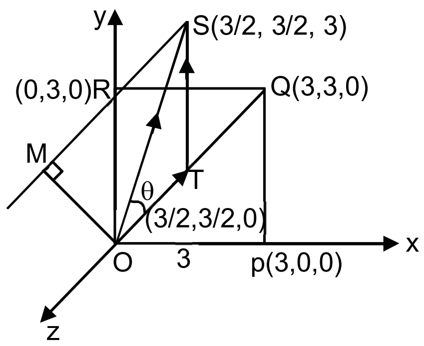
(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Ans. **[BCD]**

Sol.

(A) $OT = \sqrt{\frac{9}{4} + \frac{9}{4}} = \frac{3}{\sqrt{2}}$



$$OS = \sqrt{\frac{9}{4}+\frac{9}{4}+9} = \frac{3\sqrt{3}}{\sqrt{2}}$$

$$\cos\theta = \frac{\frac{3}{\sqrt{2}}}{\frac{3\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}}$$

0

(B) Equation of plane OQS

$$\begin{vmatrix} x & y & z \\ 3 & 3 & 0 \\ 3/2 & 3/2 & 3 \end{vmatrix} = 0 \Rightarrow x - y = 0$$

(C) Perpendicular distance of the plane $x - y = 0$ from the point P(3, 0, 0) is $= \frac{3}{\sqrt{2}}$

(D) Straight line RS

$$\frac{x-0}{\frac{3}{2}} = \frac{y-3}{\frac{-3}{2}} = \frac{z}{3} \Rightarrow \frac{x}{1} = \frac{y-3}{-1} = \frac{z}{2} = \lambda$$

Let $M \equiv (\lambda, -\lambda + 3, 2\lambda)$

OM perpendicular to RS $\Rightarrow \quad \lambda + \lambda - 3 + 4\lambda = 0 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\therefore \quad M \equiv \left(\frac{1}{2}, \frac{5}{2}, 1\right), \qquad OM = \sqrt{\frac{1}{4}+\frac{25}{4}+1} = \sqrt{\frac{15}{2}}.$

43. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Ans. **[ABC]**

Sol. $C_1 : x^2 + y^2 = 3$

$P : x^2 = 2y$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3, y = 1$

$P(\sqrt{2}, 1)$

Tangent at $P(\sqrt{2}, 1)$

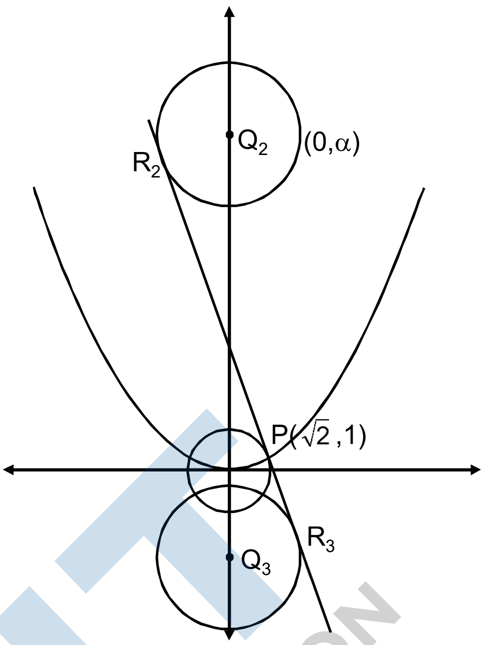
$\sqrt{2}x + y = 3$

$Q_2\ (0, \alpha),\ r = 2\sqrt{3}$

$\left|\dfrac{0+\alpha-3}{\sqrt{3}}\right| = 2\sqrt{3}$.

$\alpha = \pm 6 + 3;\ \alpha = 9, -3$

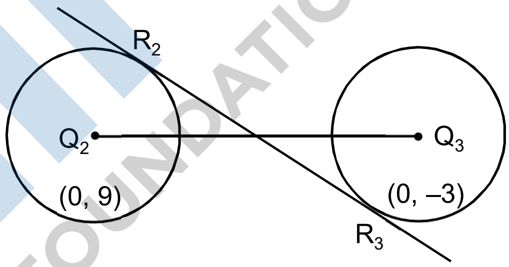
$Q_2(0, 9),\ Q_3(0, -3)$



(A) $Q_2Q_3 = 12$

(B) R_2R_3 = (length of internal tangent)

$= \sqrt{(12)^2 - \left(2\sqrt{3}+2\sqrt{3}\right)^2} = \sqrt{144-48} = \sqrt{96} = 4\sqrt{6}$



(C) area $(\Delta OR_2R_3) = \dfrac{1}{2}$ (B) (H) $= \dfrac{1}{2}\left(4\sqrt{6}\right)\left(\sqrt{3}\right) = 6\sqrt{2}$

(D) area $(\Delta PQ_2Q_3) = \dfrac{1}{2}\begin{vmatrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{vmatrix} = 6\sqrt{2}$.

44. Let $f : (0, \infty) \to R$ be a differentiable function such that $f'(x) = 2 - \dfrac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(A) $\lim\limits_{x\to 0^+} f'\left(\dfrac{1}{x}\right) = 1$

(B) $\lim\limits_{x\to 0^+} xf\left(\dfrac{1}{x}\right) = 2$

(C) $\lim\limits_{x\to 0^+} x^2 f'(x) = 0$

(D) $|f(x)| \le 2$ for all $x \in (0, 2)$

Ans. **[A]**

Sol. $f'(x) = 2 - \dfrac{f(x)}{x}$

$x f'(x) + f(x) = 2x$

$\dfrac{d}{dx}(x\, f(x)) = 2x$

$x f(x) = x^2 + C$

$f(x) = x + \dfrac{C}{x}$ $\quad f(1) \neq 1 \quad \Rightarrow \quad C \neq 0$

(A) $\lim f'\left(\dfrac{1}{x}\right) = \lim\, (1 - Cx^2) = 1 \quad \Rightarrow \quad$ (A) is correct

(B) $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right) = \lim_{x\to 0^+} \left(1+Cx^2\right) = 1$ $\Rightarrow$ (B) is incorrect

(C) $\lim_{x\to 0^+} x^2 f'(x) = \lim_{x\to 0^+} x^2\left(1-\frac{C}{x^2}\right) = -C \neq 0$ $\Rightarrow$ (C) is incorrect

(D) $|f(x)| = \left|x+\frac{C}{x}\right| = |x| + \frac{|C|}{|x|} \leq 2\sqrt{|C|}$ depends on C.

45. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = \frac{-k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(A) $\alpha = 0, k = 8$

(B) $4\alpha - k + 8 = 0$

(C) $\det(P \text{ adj } (Q)) = 2^9$

(D) $\det(Q \text{ adj } (P)) = 2^{13}$

Ans. **[BC]**

Sol. $Q = P^{-1}k$

$|Q| = \frac{k^3}{|P|} = \frac{k^2}{2}$

$|P| = 2k$

$Q = \frac{k\,(\text{adj}.P)}{|P|} = \frac{k}{|P|}$ adj. (P) $= \frac{-k}{|P|}(3\alpha+4) = \frac{-k}{8}$

$\Rightarrow 24\alpha + 32 = |P|$

$|P| = 12\alpha + 20$

$24\alpha + 32 = 12\alpha + 20 \Rightarrow \alpha = -1$

$|P| = 8, k = 4$

(B) $4\alpha - k + 8 = 0$

(C) $|P \text{ adj. } (Q)| = |P||Q|^2 = 8 \cdot 8^2 = 2^9$

(D) $|Q \text{ (adj. P)}| = |Q||\text{adj. P}| = |Q| \cdot |P|^2 = 8^3 = 2^9$.

46. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$

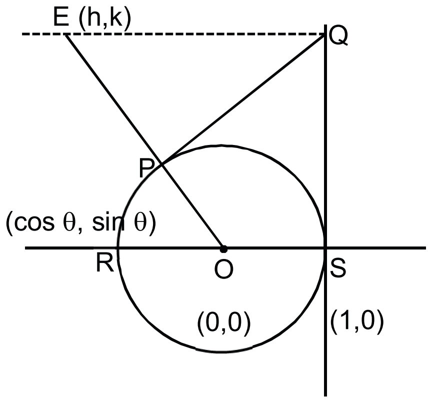
(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, \frac{-1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{4}, \frac{-1}{2}\right)$

Ans. **[AC]**

Sol. RS : $y = 0 \rightarrow$ x-axis



Let P (cos θ, sin θ)

Tangent at S (1, 0) : x – 1 = 0

Tangent at P (cos θ, sin θ) : xcos θ + ysin θ = 1

Intersection: x = 1, $y = \dfrac{1-\cos\theta}{\sin\theta}$

$\therefore \quad Q\left(1, \dfrac{1-\cos\theta}{\sin\theta}\right)$

Normal at P

$y = \tan\theta \cdot x$ and $y = \dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$

$$y = \left(\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)x = \left(\frac{2y}{1-y^2}\right)x$$

$y = 0,\ y^2 = 1 - 2x.$

Check by option A, C

47. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\dfrac{dy}{dx} - y^2 = 0,\ x > 0$, passes through the point (1, 3). Then the solution curve

(A) intersects y = x + 2 exactly at one point

(B) intersects y = x + 2 exactly at two points

(C) intersects $y = (x + 2)^2$

(D) does **NOT** intersect $y = (x + 3)^2$

Ans. **[AD]**

Sol. $\left((x+2)^2 + y(x+2)\right)\dfrac{dy}{dx} - y^2 = 0$

Method-1: $\dfrac{dx}{dy} = \dfrac{(x+2)^2}{y^2} + \dfrac{(x+2)}{y}$

$$\frac{1}{(x+2)^2}\cdot\frac{dx}{dy} = \frac{1}{y^2} + \frac{1}{y(x+2)}$$

Let $\dfrac{1}{x+2} = t$

$$\frac{-1}{(x+2)^2}\frac{dx}{dy} = \frac{dt}{dy} \Rightarrow \frac{-dt}{dy} - \frac{t}{y} = \frac{1}{y^2} \Rightarrow \frac{dt}{dy} + \frac{t}{y} = \frac{-1}{y^2}$$

$$t \cdot y = -\int\frac{dy}{y} + C$$

$$\frac{y}{x+2} = -\ell n\, y + C$$

$\frac{3}{3} = -\ell n\, 3 + C \Rightarrow C = 1 + \ell n\, 3.$

$\frac{y}{x+2} = -\ell n\, y + 1 + \ell n\, 3$

$\frac{y}{x+2} = \ell n\left(\frac{3e}{y}\right) \Rightarrow y \cdot e^{\left(\frac{y}{x+2}\right)} = 3e$

Method-2: $\frac{dy}{dx} = \frac{y^2}{(x+2)^2 + y(x+2)} \Rightarrow \frac{dx}{dy} = \frac{(x+2)^2 + y(x+2)}{y^2}$

$\frac{dt}{dy} = \frac{t^2 + ty}{y^2}$

$t = vy$

$v + y\frac{dv}{dy} = v^2 + v$

$\frac{dv}{v^2} = \frac{dy}{y}$

$\frac{1}{v} = -\ell n\, y + C$

$\frac{y}{x+2} = -\ell n\, y + C$

$\frac{3}{3} = -\ell n\, 3 + C \Rightarrow C = 1 + \ell n\, 3.$

$\frac{y}{x+2} = -\ell n\, y + 1 + \ell n\, 3$

$\frac{y}{x+2} = \ell n\left(\frac{3e}{y}\right) \Rightarrow y \cdot e^{\left(\frac{y}{x+2}\right)} = 3e.$ $\Rightarrow$ (A) & (D) correct.

48. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Ans. **[ACD]**

Sol. $2s = x + y + z$

$\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ = k (let)

$$\left.\begin{array}{l} -x+y+z=8k \\ x-y+z=6k \\ x+y-z=4k \\ \hline x+y+z=15k \end{array}\right] \Rightarrow x = 5k,\ y = 6k,\ z = 7k$$

Add

$\therefore \Delta = \sqrt{9k \cdot 4k \cdot 3k \cdot 2k} = k^2 6\sqrt{6}$

$r = \frac{\Delta}{s} = \frac{k^2 6\sqrt{6}}{9k} = \frac{2}{3}\sqrt{6}$ k.

area of incircle = $\frac{8\pi}{3}$

$\pi\left(\frac{2}{3}\sqrt{6}\right)^2 \cdot k^2 = \frac{8\pi}{3} \Rightarrow k = 1$

$\therefore$ x = 5, y = 6, z = 7

(A) area (Δxyz) = $k^2 6\sqrt{6} = 6\sqrt{6}$

(B) $R = \frac{abc}{4\Delta} = \frac{5 \cdot 6 \cdot 7}{4 \cdot 6\sqrt{6}} = \frac{35}{4\sqrt{6}}$

(C) $\sin\frac{X}{2} \cdot \sin\frac{Y}{2} \cdot \sin\frac{Z}{2} = \frac{r}{4R} = \frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{1-\cos(X+Y)}{2} = \frac{1+\cos Z}{2} = \frac{1+\frac{1}{5}}{2} = \frac{3}{5}$ $\left(\because \cos Z = \frac{1}{5}\right)$.

49. Let f: R $\to$ R, g: R $\to$ R and h: R $\to$ R be differentiable functions such that f (x) = $x^3 + 3x + 2$, g(f(x)) = x and h(g(g(x))) = x for all x $\in$ R. Then

(A) $g'(2) = \frac{1}{15}$ (B) h'(1) = 666 (C) h(0) = 16 (D) h(g(3)) = 36

Ans. **[BC]**

Sol. Given f(x) = $x^3 + 3x + 2$, $g(f(x))$ = x and $h(g(g(x)))$ = x

(A) $\because g(f(x)) = x \Rightarrow g'(f(x)) \cdot f'(x) = 1 \Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

f(x) = 2 when x = 0

$\Rightarrow g'(2) = \frac{1}{f'(0)} \Rightarrow g'(2) = \frac{1}{3}$

(B) $h'(g(g(x))) \cdot g'(g(x)) \cdot g'(x) = 1$

$\because g(g(x)) = 1 \Rightarrow g(x) = f(1) = 6 \Rightarrow x = f(6) = 236$

$\therefore h'(1) \cdot g'(6) \cdot g'(236) = 1$

$\therefore g'(6) = \frac{1}{f'(1)} = \frac{1}{6}$ and $g'(236) = \frac{1}{f'(6)} = \frac{1}{111}$

$\Rightarrow h'(1) \times \frac{1}{6} \times \frac{1}{111} = 1 \Rightarrow h'(1) = 666$

(C) $\because h(g(g(x))) = x$

$g(g(x)) = 0 \Rightarrow g(x) = f(0) = 2 \Rightarrow x = f(2) = 16$

$\therefore h(0) = 16$

(D) $\because h(g(g(x))) = x$

$\therefore g(x) = 3 \Rightarrow x = f(3) = 27 + 9 + 2 = 38$

$\therefore h(g(3)) = 38.$

SECTION 3

50. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

Ans. **[1]**

Sol. Consider $f(x) = \int_0^x \frac{t^2}{1+t^4} dt - (2x-1)$

$f'(x) = \frac{x^2}{1+x^4} - 2 = \frac{1}{x^2 + \frac{1}{x^2}} - 2 < 0 \ \forall x \in [0, 1]$

$\Rightarrow$ f (x) is decreasing function

Now f (0) = 1 and $f(1) = \int_0^1 \frac{t^2}{1+t^4} dt - 1 = -ve$

$\Rightarrow$ exactly one solution of f (x) = 0.

51. Let $\alpha, \beta \in R$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

Ans. **[7]**

Sol. $\because \lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$

$\Rightarrow \lim_{x \to 0} \frac{\beta x^3}{\alpha x - \left(x - \frac{x^3}{3!} \ldots\ldots\right)} = 1$

for finite non-zero limit. $\alpha - 1 = 0 \Rightarrow \alpha = 1$

and limit $6\beta = 1$

52. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\ {}^{51}C_3$ for some positive integer n. Then the value of n is

Ans. **[5]**

Sol. $\because (1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$

$$\Rightarrow (1 + x)^2 \frac{\left((1+x)^{48} - 1\right)}{(1+x)-1} + (1 + mx)^{50} = \frac{1}{x}\left((1+x)^{50} - (1+x)^2\right) + (1 + mx)^{50}$$

$\therefore$ coefficient of $x^2 = {}^{50}C_3 + {}^{50}C_2 \cdot m^2 = (3n + 1) \cdot {}^{51}C_3$

$$\Rightarrow \frac{50 \times 49 \times 48}{6} + \frac{50 \times 49}{2} m^2 = (3n + 1) \cdot \frac{51 \cdot 50 \cdot 49}{6}$$

$$\Rightarrow 8 + \frac{m^2}{2} = (3n + 1) \cdot \frac{17}{2}$$

$\Rightarrow m^2 = 51n + 1$

for n = 5, $m^2 = 256 \Rightarrow m = 14$

$\therefore$ n = 5. **Ans.**

53. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

Ans. **[2]**

Sol. $\because \begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 2x & 4x^2 & 8x^3 \\ 3x & 9x^2 & 27x^3 \end{vmatrix} = 10$$

$$\Rightarrow x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$\Rightarrow x^3 \{1 (-5) - 1 (-1) + 1 (6)\} + x^6 \{1 (36) - 1 (30) + 1 (6)\} = 10$

$\Rightarrow 2x^3 + 12x^6 = 10$

$\Rightarrow 6x^6 + x^3 - 5 = 0 \Rightarrow (x^3 + 1)(6x^3 - 5) = 0$

$$\Rightarrow x^3 = -1 \text{ or } \frac{5}{6}$$

$\therefore$ Two real solutions for x.

54. Let $z = \frac{-1+\sqrt{3}i}{2}$, wherre $i = \sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

Ans. [1]

Sol. $Z = \frac{-1+\sqrt{3}i}{2} = \omega$

$$p = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix} = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$\therefore$ $$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} (-\omega)^{2r} + \omega^{4s} & (-\omega)^{r+2s} + \omega^{r+2s} \\ (-\omega)^{r+2s} + (\omega)^{r+2s} & \omega^{4s} + \omega^{2r} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\Rightarrow$ r + 2s should be an odd integer.

$\therefore$ r should be odd integer

and $\omega^{2r} + \omega^{4s} = -1$

$\Rightarrow$ $1 + \omega^{2r} + \omega^{4s} = 0$

$\therefore$ r = 1 and s = 1 are the only values satisfying it.

$\therefore$ Number of ordered pairs = 1.